

# 12 GOOD <br> PRACTICES TO LEARN MATHS 

"Become European Math Champions"<br>2020-1-ES01-KA229-082183<br>CROATIA - GREECE - ITALY<br>PORTUGAL - ROMANIA - SPAIN



CROATIA
Measuring and recycling ..... 1
Triangle constructions ..... 11
GREECE
Playing with fractions ..... 16
Physical Education and maths ..... 19
ITALY
Building Geometric Solids ..... 21
Paper party ..... 24
PORTUGAL
Number decomposition: addition ..... 28
Sequences and regularities ..... 31
ROMANIA
Maths kingdom through contest ..... 33
Geometry ..... 35
SPAIN
Dividing through decomposition ..... 37
Dividing through Cuisenaire rods ..... 39

# MEASURING AND RECYCLING 

## MAKING BAGS AND DECORATING WITH TANGRAM SHAPES

EXPLANATORY VIDEO:

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HO1BDVWNOX5TXRPMYLZYSB_DATGUO/VIEW?
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## LEARNING OBJECTIVES:

- Measurement and calculation of the extent and surface of the created shape,
- Discussion about the accuracy of measurement results and calculations,
- Making bags,
- Learning through playing,
- Discussion of the relationship between the surfaces of individual parts of the tangram,
- Discussion about the usefulness of what has been done and what has been learned.


## TOPIC:

Measuring and calculating the perimeter and area of geometic shapes

## ACTIVITY:

Recycling - making shopping bags and decorating them using tangram shapes
STUDENT AGE: 5th grade (approx. 11 years old)

ACTIVITY DURATION: 90 minutes, possibility to finish the bag with the housemates

## MATERIAL NEEDED FOR IMPLEMENTATION:

- T-shirts that students have outgrown
- thread
-scissors
-tangram
- tangram shape templates


## SKILLS THAT STUDENTS DEVELOP:

- measuring the length of the sides of an individual geometrical shape using a geometric tool
- calculation of the perimeter and area of the shape
- conversion of measurement units
- shaping the bag
-tying knots
- cutting and shaping fabric (scissors)
- sewing (gluing)
-organizing stages of work
- taking notes during research
-critical analysis of scientific research results
- critical thinking about the recycling method


## ACTIVITY 1

Students repeat what they know about the tangram puzzle and agree on the rules for
making the bag.
Tangram is a traditional Chinese puzzle, made from different materials.
It consists of seven parts - geometric shapes:
five triangles, squares and parallelograms.


The stacking possibilities are unlimited, but according to certain simple rules. Students can combine different motifs
When arranging one motif:
a) all forms must be used
b) all shapes must touch
c) shapes must not overlap.


## ACTIVITY 2-GLUING/SEWING TANGRAMS

Students should bring old T-shirts and according to the instructions and the video bags will be made: https://youtu.be/WaJQfXOjAE4 The 1st step is shaping the bag:


2nd step - choosing the character that will be the decoration on the bag


Step 3 - cutting the tangram shapes from the canvas, stacking the chosen shape from the tangram shape and gluing them to the bag. Those students who wish can secure the pasted shape by sewing the edges of the shape.


## Some of the selected shapes:



## ACTIVITY 3 - CALCULATION

After the gluing/sewing shapes, there are tasks for the students:

1) measure the outer lines of the shape and record the measurement
2) calculate the perimeter in centimeters ( $o=a+b+c+d+e+\ldots$ )
3) convert that perimeter into millimeters, decimeters and meters.

4) Calculate the area of the resulting shape. $A=(a \cdot b) / 2$

Before you start, compare the areas of individual parts of the tangram!


- Uporedimo površine svih dijelova tangrama!
- Neka je površina najmanjih trokuta jednaka P...


Conclusion: it is enough to calculate the area of the smallest triangle and multiply it 16 times in order to get the area of the shape which is composed of 7 tans (parts of the tangram).

## WE CAN ALSO DO:

It is also possible to arrange different geometric shapes of equal areas.


- Svi likovi saștoje se od istih dijelova, pa zaključujemo da su njihove površine
- jednake!



When arranging given motifs, geometric shapes are learned and adopted, size and shape relationships, different positions are observed. The application of mathematical formulas for the perimeter and area of the geometric shapes is also visible.
By continuous arranging, a better ability to visualize objects and spatial relationships in the environment is developed as one of the prerequisites for coping, creativity and creativity in all areas of human activity and expression. Environmental awareness and thinking about the possibilities of recycling are stimulated.
*According to the idea: ecko@eckong.com and http://www.antonijahorvatek.from.hr/

## TRIANGI.E CONSTRUCTIONS CROATIA

## TRIANGLE CONSTRUCTIONS

## CHRISTMAS TREE - THREE BASIC TRIANGLE CONSTRUCTIONS

EXPLANATORYVIDEO:
HTTPS://DRIVE.GOOGLE.COM/FILE/D/1VN90VSJ UHSEEI8MFOS2HTXMIUUVVBNBH/VIEW? $\underline{U S P}=S H A R I N G$

## LEARNING OBJECTIVES:

- Construct triangles, analyzes their properties and relationships.
- Remind students of the importance of drawing a sketch and marking known quantities in a triangle.
- Underlining each step, the teacher demonstrates the construction of the SSS on the board, and the students construct the triangle in the notebook.


## STEPS TO FOLLOW

## 1. Let's repeat

Activity outcome: sketches and constructs triangles according to SSS, SKS and KSK lessons on conformity.
Description of the activity:
Divide the students into 6 non-homogeneous groups of 4 students each and choose group leaders who will take care of the working atmosphere in the group.

## 2. Solving tasks

Outcome of the activity: sketches and constructs triangles according to SSS, SKS and KSK instructions on conformity; adapts and harmonizes his behavior with the rules in the group; demonstrates the skills of agreement, negotiation and compromise; explains the importance of cooperation. recognizes the features of assertive behavior; talks about the benefits of teamwork for everyone; assumes the assigned role in the team, actively contributes with his participation; applies presentation skills.

## Description of the activity:

- Students solve the tasks: 1. to 5.

Students within the group solve all tasks, compare procedures and solutions, help each other, and the group leader controls the work. The teacher goes around the students and checks the accuracy of the solutions. At the end, the students hand in the finished work with a "Christmas tree"


Konstrukcija SKS: stranica - kut - stranica
Construction: page - angle - page


1. Draw a length $\overline{A B}$ long $c$.
2. At the top of $B$ construct- make an angle of $45^{\circ}$.
3. Measure the length a from the point $B$ along the side of that angle - this gives us the point $C$.
4. Connect the tops.

5. Assignments
6. Paper on which the Christmas tree will be arranged with a constructed isosceles triangle with a base length of 40 cm and a leg length of 28.3 cm (the picture on the right shows how the triangles that the students cut out should be glued to the base. an isosceles triangle).
7. Green paper lit would be good if it was colored on both sides because of the stacking of the Christmas tree.
8. Collage paper in color and various decorations.
9. Scissors and glue.




Konstrukcija SSS: stranica - stranica - stranica
Construction: page - page - page

1. Apply a length of $a$.
-Konstrukcija SSS:
2. From the left end point, make port of a circle with radius $b$.
3. From the right end point, moke part of a circle with radius $c$.
4. The place where the circles
infersect is actually the third
intersect is actually the third
the tops to each other.


Konstrukcija KSK: kut - stranica - kut
Construction: angle - page - angle

1. Drow a length $\overline{A B}$ long $c$.
2. Construct a $30^{\circ}$ angle at fop $A$.
3. Construct a $60^{\circ}$ angle at top $B$.
4. The place where the legs of those angles (red and purple) intersect is the third vertex $C$. Connect $A, B$ and $C$
~Konstrukcija KSK:
$\cos \mathrm{cm} \quad a=30^{\circ} \quad \beta=60^{\circ}$


WORK INSTRUCTIONS:
d

1. Solve all tasks in a notebook, respecting the rules of group work.
2. Divide the tasks between each other so that each student constructs at least two triangles on green paper, but so that all toriks are solved.
3. Cut out all the constructed triangles.
4. Assemble a Chistmas tree (pine) from the triangles and glue the triangles this assembled on a large piece of paper with a sketch of the Chriatmas tree.
5. Decorate the Christmas tree as desired.




## PLAYING WITH FRACTIONS

GREECE

# Playing with fractions 4th Primary school of Levadia Erasmus+Become European Math Champions 

EXPLANATORYVIDEO:
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## LEARNING OBJECTIVES:

- Explain why one fraction is equivalent to another fraction using visual representations.
- Recognize and produce equivalent fractions.
- Compare fractions with the same numerator and the same denominator.
- Recognize that comparisons are made only when fractions refer to the same whole.
- Compare fractions with different numerators and denominators by creating common numerators or denominators by comparing each:
- with visual representations where e.g. it will be stated that track $1 / 3$ is different from track 1/7,
- using arithmetic strategies (after having practiced enough with the previous ones) e.g. comparing 5/6 and 9/12 to convert 5/6 to the equivalent of $10 / 12$ and compare this with $9 / 12$ since they have the same denominators.


## Prerequisite knowledge:

- Place fractions on the number line.
- Be able to represent fractions larger than one (mixed numbers).
- Construct visual representations to express what a fraction shows.
- To compare fractions symbolically and verbally.


## Conclusions:

- A number can be written in different forms without losing its value.
- Comparison of fractions can be done either visually (visual representations) or using mathematical strategies.
- Two fractions do not have to be the same to be equivalent.
- Between two fractions there are infinite fractions.
- Fractions are numbers and express part of the whole. Therefore it must always be understood in order to be evaluated in relation to the place.


## Stage 2 - Evaluation

Students will be able to:

- construct equivalent fractions by multiplying or dividing (simplifying)
numerator and denominator by the same number.
- identify equivalent fractions with the help of visual representations.
- solve and construct problems with fraction equivalence.
- compare (knowing that the comparable fractions refer to the same whole) fractions of the same name, in the number line taking as the whole $1 / 2$ or the unit, with the help of visual representations and using strategies to convert the heteronymous fractions into homonymous ones.


## Stage 3 - Learning plan / Learning activities

Regarding fractions, after they have been taught in a previous class we ask if they remember them. Brief reminder of the terms of a fraction.
It is given on paper a representation of a pizza and they are called to an activity of breaking it into pieces.

## Exercise 1:

Problem reported:
Danae and Lefteris bought two identical pizzas. Danae ate $3 / 6$ of the pizza and Lefteris ate $3 / 9$. They both believe that they ate an equal amount of pizza. They are right; Justify your answer with representation and calculations.
In this exercise both children eat the same number of pieces, but they have to think about whether they eat the same amount of pizza. Most students will think that they are eating an equal amount of both, without taking into account that one pizza is divided into fewer pieces and the second into more (so in the first the pieces will be larger than in the second).

## Exercise 2:

In this exercise students are given pairs of fractions to compare. The children are asked to find on their own the mathematical strategy based on equivalence to compare fractions. They draw the part of the whole and find that different fractions belong to the same piece (equivalent Fractions).
This equates equivalence with comparison.
They then place fractions next to a pizza design that is divided each time into a different number of pieces.

## LEARNING OBJECTIVES:

- Purpose: Approach to mathematics - geometry - through the course of Physics Education for 3 rd and 4th grade elementary school children.
- Objective: The creation of the basic geometric shapes with their bodies children in groups.


## STEPS TO FOLLOW:

- Students are divided into groups and asked to form with their bodies at first straight lines, curved and checkered to practice cooperation between them.
- The same body shape is required to be performed each time which they will choose from all participants in the group (eg all standing or all squatting ).
- Students are then asked to create with their bodies geometric shapes (circle, square, triangle, etc.)
- The team that will be the first to form the shape that will be requested each time wins.


# BUILD GEOMETRIC SOLIDS 

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## LEARNING OBJECTIVES:

- Use appropriate geometric vocabulary to describe properties and attributes of two- and three - dimensional figures.
- examine, recognize, name and explore the properties of geometric solids.
- Identify geometric solids in the world around them.
- raw geometric solids using the technique of orthogonal projections.


## Vocabulary:

Geometry, Solid, Sphere, Cylinder, Cone, Cube, Rectangular Prism, Triangular Prism

## Materials Needed:

- Patterns of geometric solids to cut out and build.


## Activity:

1. In pre-arranged cooperative groups, let students handle, examine and discuss the physical characteristics of the geometric solids.
2. Each student will draw geometric solids using the technique of orthogonal projections. They will compare the solids they draw.
3. As a class, generate a list of characteristics for each solid. Record the information on a chart. (Ex: Which one roll? Which have corners, sides and bases?)
4. Teacher will then display some physical models of geometric solids to the class that are seen in the world around us daily. Students will identify the names of the geometric solids.
5. Starting from the patterns of geometric solids, students will cut and build the 3D solids.

## Assessment:

The students will bring in examples, draw or glue a picture and label two of each type of geometric solid examples. They must be prepared to state where this solid can be found in the real world.

## Teaching Notes:

In this lesson students will be able to use appropriate geometric vocabulary to describe properties and attributes of two and three dimensional figures. They will also be able to recognize, name and explore the properties of geometric solids. Finally, students will identify geometric solids in the world around them.
It is very important to use a variety of supplementary materials while teaching new material to students in order to make the concepts comprehensible. While teaching the activity, I would use hands-on manipulatives, realia, pictures and visuals while teaching. These teaching aids will assist all learners, especially ESL and ESE students, in comprehending the materials.

First, wooden geometric solids would be hands-on manipulatives for students to use. Hands-on learning helps all learners relate to the new information better. Realia, real life objects, should be brought in to demonstrate to students that items in your house or that you see daily are considered geometric solids too. They have the similar attributes. Pictures of various real-life buildings and items can be showed to the class of where these geometric solids are seen daily. Lastly, while listing all of the physical characteristics of each solid, we would make a detailed, visual chart as a class. This will provide a better understanding when processing these concepts.

Overall, teachers must support every type of learner in their class. There is a diversity of learners in each class and each learner has a different type of learning style. Some, perhaps, may not even know the English language. Making information comprehensible to all students is crucial and all teachers should implement supplementary materials in their daily lessons.

## PAPER PARTY



EXPLANATORYVIDEO:

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## LEARNING OBJECTIVES:

- Students investigate the paper-making process by trying it themselves. Students are thrilled to find that they can make paper and that their product is practical, as well as beautiful.
- Videos of the paper-making process can also be watched..


## Subjects

Science, Technology, Social Studies, Language Arts, Visual Arts

## Time Considerations

Preparation: 30 minutes plus time to gather materials Activity: 3 sessions of 60 minutes

## Materials

- Scrap paper torn into $2.5 \mathrm{~cm} \times 2.5 \mathrm{~cm}$ pieces (paper towels, construction paper and toilet
- paper work well)
- Inox frame round ( 18 cm diameter)
- Tow plastic basin at least 8 liters in capacity, that is larger than the frame
- Cloth dishtowels (felt, blotting paper, or newspaper may substituted)
- Blender
- Sponge
- Strainer
- Absorbent cloths
- Rolling pin
- Colored paper
- Pieces of colored thread, or dried flowers (optional)


## Introduction

Paper is a simple material. It is essentially a mat held together by a fiber's roughness and can be made from almost any fibrous material such as cotton, hemp, flax, wood or recycled paper. And yet, this simple product has a tremendous effect on our lives. Imagine how different your day would be without paper!
We use paper for countless things in our everyday lives, including newsprint, magazines, schoolbooks, photocopies, computer printers, envelopes, stamps, tissue and sanitary products, bags, boxes, containers, food packaging, gift wrap, wallpaper, disposable dishes, lampshades, and as an art medium. Industrial uses include gaskets, speaker cones, liquid and gas filters, insulation, and friction devices.
The process for making paper was invented in China in the second century A.D., and all paper was made one sheet at a time until 1798. With the Industrial Revolution and the papermaking machine, papermaking became a major industry that provides countless products, from books and newspapers to packaging and note pads. Some modern machines can make a sheet of paper 8
8.8 cm wide and nearly 64 km long in just one hour! While the technology has changed dramatically over the centuries, the basic steps are simple enough for your students to do in class. The process begins when trees, grown especially for papermaking, are harvested and transported to a paper mill. At the mill, large machines strip away bark and shred the logs into millions of chips the size of breakfast cereal. The wood chips travel on conveyors to gigantic "pulp cookers," where chemicals and steam are added. The mixture is heated and pressurized, breaking the chips into smaller and smaller pieces and finally forming a dilute water suspension of wood fibers called pulp. The pulp then passes through cleaners and screens and sometimes goes through a bleaching process that will give it the whiteness needed for the grade of paper being manufactured. Other chemicals such as dyes, pigments, sizings, or resins are sometimes added to provide the paper or paperboard (thick paper for boxes) with the appropriate finish. The pulp is then pumped through pipes to a paper machine where it is sprayed onto a wide, moving wire screen. After the water in the pulp drains through the holes, a damp mat of wood fibers remains; this is the paper. It is picked up from the end of the moving belt and dried over steam-heated rollers. Most paper comes from trees, a renewable resource. Most of the trees used for paper are managed for that purpose. In commercial papermaking, more than half of the fiber comes from lumber mill residues and from paper that has been collected for recycling. Paper is easily recycled, which helps reduce waste. However, no matter how much paper we recycle, new trees still will be needed for paper products, because paper cannot be recycled indefinitely. Each time paper goes through the manufacturing process, the fibers deteriorate. After repeated recycling, the fiber is no longer suitable for papermaking. Producing recycled paper requires about $40 \%$ less energy than producing non-recycled paper. And making recycled paper produces $70 \%$ less water pollution and much less air pollutants, such as greenhouse gasses, particulates, and other hazardous pollutants.

## Activity Instructions

1. Introduce the activity by asking students what they think paper is made of and how it is made.
2. Fill the blender halfway with warm water, then add a handful of the soaked paper. Blend at medium speed until you no longer see pieces of paper, and the pulp has a soupy consistency. You can blend in a piece of construction paper for color; or stir in short pieces of thread, dried flowers, or herbs for texture.
3. Pour the mixture into the large basin and then fill the basin with warm water, mixing thoroughly until the ingredients are evenly dispersed. Adding a few ounces of liquid starch will help make the paper firm.
4. Slide the deckle into the basin. Put some pulp onto the screen and, still holding the deckle underwater, gently move it back and forth to get an even layer of fibers on the screen.
5. Lift the deckle out of the mixture, keeping it flat. Allow it to drip until most of the water has drained off. You should have a uniform layer of pulp mixture on the deckle. Press the pulp gently with your hand to squeeze out excess moisture (rubber gloves will help). Soak up any excess water from the bottom of the screen with a sponge.
6. Place newspaper on a flat surface and turn the screen paper-side-down on the cloth. Lift the screen gently, leaving the paper. Gently tap the screen to help release the paper.
7. Let it dry naturally for several hours or overnight. Gently peel off the paper when it is dry.
8. When you're finished making paper, collect the leftover pulp in a strainer and recycle it, or freeze it in a plastic bag for future use. Don't pour the pulp down the drain!

## DECOMPOSITON NUMBER - ADDITION <br> PORTUGAL

## LEARNING OBJECTIVES:

- Learn a decomposition of numbers (number 5)
- Using concrete material in learning mathematics
- Associating mathematics with concrete situations


## STEPS TO FOLLOW:

- We'll see different possibilities of getting the number 5 by adding
- We will use bottle caps of different colors
- Each digit will be represented by a different color
- Let's put these caps together of different colors so we can get 5 caps
- Represent in your notebook through mathematical symbols the different shapes you used to get 5
- Ex: $2+1+2=5$
- Represent in your notebook through a graph the different shapes you used to get 5




# SEQUENCES AND REGULARITIES 



## LEARNING OBJECTIVES:

- Sort objects according to a sequence/regularity
- Articulating mathematics with digital educational resources


## STEPS TO FOLLOW:

- In class students will sort objects according to certain characteristics (color, shape, size, theme,...)
- Then an App will be used where students on the computer will complete different sequences of images



## MATHS KINGDOM

EXPLANATORYVIDEO:

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LEARNING OBJECTIVES:

- Revise maths concepts through a contest.


## STEPS TO FOLLOW:

The class is divided in 2 groups

## Activity 1

- A dice is thrown and a number is obtained.
- The pupils do the sum between the square and the cube of the number.
- The pupils draw a square with the side ncm , The, they calculate the area and the perimeter of the square.
- The pupils draw a circle with the circle's radius n cm.


## Activity 2

- Two dices are thrown and two numbers are obtained: n and m .
- The pupils calculate $m$ to $n$th $a n d n$ to the mth.
- The pupils calculate the sum of the number bewteen mn si dnm.
- The pupils write the fractions $m$ over $n$ and $n$ over $m$. Then, they calculate the sum, the difference, the product and the quotient of the fractions.
- The pupils dra a rectangle with the sides n and m cm and calculate the area and the perimeter of the triangle.


## Acitivity 3

- Three dices are thrown and three numbers are obtained; $n, m$ and $p$.
- The pupils write all the posible subunit fractions, using 3 numbers.
- The pupils write all the number: $m, n, p$ and they order them descending


## GEOMETRY

ROMANIA



EXPLANATORYVIDEO:

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## LEARNING OBJECTIVES:

- to use correctly the maths terminology: (triangle, square, circle, rectangle, cube, cuboid);
- to manifest a team spirit throughout the activity, but to act independently when achieving the tasks, too.
- to understand the utility of geometric shapes in our daily lives and to create objects using them.


## stages of the lesson:

- The students have been divided in teams. Each team was responsible for fulfilling a task.
- One of the teams made models using the Tangram technique, another one used sticks in the shape of geometric shapes in order to build things.
- The students collaborated in order to fulfil the tasks they received, had a good time and understood the usefulness of studying geometric shapes. They very much appreciated the practical jobs, as well as connecting maths to real life situations.
- As a result, the students grew more confident and realised they CAN be a part of their of the learning process.


# DIVIDING THROUGH DECOMPOSITION 



EXPLANATORYVIDEO:

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## LEARNING OBJECTIVES:

- Learn the concept of division through decomposition.
- Divide decomposing numbers by representing it visually usind the "spider method".
- Improve the mental calculus of the students.


## STEPS TO FOLLOW:

- Explain the meaning of dividing and sharing.
- We chose an amount X (dividend) like in the example in order to divide by one divisor $X$.
- We have to decompose the dividend and divide these numbers decomposed by the divisor $X$.
Ex:
345: $5=$
300:5=60
$40: 5=8$
5:5 = 1
Result: 69

The last step is to sum the results of the divisions that we have done previously.

## DIVIIING USING CUISENAIRE RODS

EXPLANATORYVIDEO:

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## LEARNING OBJECTIVES:

- Learn the concept of division through decomposition.
- Divide using manipulative materials such as Cusinaire Rods


## STEPS TO FOLLOW:

- Choose a number and representate it using cursinaire rods.
- Take the number of trays needed according to the parts the number is going to be divided.
- The number that has to be divided is decomposed in smaller parts by using Cusinaire rods by showing how many smaller quantities are equivalent to one larger quantity.
- Once, the descomposition is done, the number can be divided using the trays. If necessary, the number can be decomposed again in smaller parts until the division is finished.


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